Eur. Phys. J. C 44, 591–598 (2005)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02375-9

THE EUROPEAN
PHYSICAL JOURNAL C

On superfield covariant quantization in general coordinates

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Received: 23 June 2005 / Revised version: 18 July 2005 /
Published online: 6 October 2005 –

Abstract. We propose a natural extension of the BRST–antiBRST superfield covariant scheme in general coordinates. Thus, the coordinate dependence of the basic tensor fields and scalar density of the formalism is extended from the base supermanifold to the complete set of superfield variables.

1 Introduction

The principle of extended BRST symmetry applied to general gauge theories has resulted in various schemes of covariant quantization [1–3]. It turns out that these schemes can be combined within the formalism [4–6] which realizes the modified triplectic algebra [3] in general coordinates. The differential operators $\varDelta^a$, V^a, U^a that form this algebra are constructed [6] in terms of a non-degenerate antisymmetric tensor ω_{ij}, a symmetric tensor g_{ij} and a scalar density ρ, defined on a supermanifold with a symmetric connection (Christoffel symbols). In Darboux coordinates, this supermanifold (*base supermanifold*) is parameterized by the fields and antifields $\left(\phi^A, \bar{\phi}_A\right)$ used in the quantization schemes [1–3]. It proves possible to fulfill the relations of the modified triplectic algebra in case the tensor field ω_{ij} endows the base supermanifold with a symplectic structure respected by the symmetric connection (covariant derivative). In this sense, the base supermanifold can be regarded as a Fedosov supermanifold [6], which generalizes the notion of Fedosov manifolds [7]. The properties of such supermanifolds have recently been studied in [8].

In the original work [4] on the modified triplectic quantization in general coordinates, the authors raised the problem of a superfield description of their formalism. This task calls for an extension of the geometric contents of [4–6] to the complete set of variables of [1–3], which can be regarded as superfield components in a superspace with a pair of anticommuting coordinates [9]. At present, two different approaches [10, 11] to the mentioned problem have been proposed. In [10], a superfield description of $\varDelta^a$, V^a, U^a is suggested, using a covariant differentiation in terms of superfield variables. This formalism leaves intact the basic ingredients of [4–6] as functions on the base supermanifold (accordingly interpreted as a Fedosov supermanifold). On the contrary, in [11] it is proposed to extend the structure of a Fedosov supermanifold to the superfield case. It turns out, however, that the resulting Christoffel symbols [11] cannot be regarded as connection coefficients having the correct properties under coordinate transformations. Thus, the approach [11] faces difficulties.

In this respect, the aim of the present work is to examine an alternative extension of the superfield approach [10]. Namely, we consider extended counterparts of the objects ω_{ij}, g_{ij}, ρ, defined on the complete supermanifold of variables [1–3], and realize the operators subject to the modified triplectic algebra in terms of such functions. In the limit when the coordinate dependence of the mentioned functions is restricted to the base supermanifold, one recovers the structure of a Fedosov supermanifold, and the present quantization scheme reduces to that of [10].

2 Basic objects

In this section, we recall the basic ingredients of [10], namely, the notion of a base supermanifold, the related construction of a *triplectic supermanifold*, and its superfield formulation. In particular, we recall the basics of tensor analysis on supermanifolds (for details, see the monograph [12] and papers [6, 8]). We use DeWitt's condensed notation [13] and designations adopted in [10]. Left-hand derivatives are denoted by $\partial_i A = \partial A / \partial x^i$, and right-hand derivatives are labeled by the subscript "r", with the corresponding notation $A_{,i} = \partial_r A / \partial x^i$. We assume that covariant derivatives, ∇, and other operators defined on tensor fields *act from the right:* $A\nabla$; besides, when necessary, the action of an operator from the right is indicated by an arrow: $\overleftarrow{\nabla}$. The Grassmann parity of a quantity A is denoted by $\epsilon(A)$.

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2.1 Triplectic supermanifold

Let us consider a supermanifold M, $\dim M = N = 2n$, with local coordinates (x^i), $\epsilon(x^i)=\epsilon_i$. We now extend M to a supermanifold $\mathcal{M}$, $\dim\mathcal{M}=3N$, with local coordinates (x^i,θ^i_a), where the additional coordinates θ^i_a are combined into $Sp(2)$-doublets (labeled by the index $a=1,2$) and possess the Grassmann parity, $\epsilon(\theta^i_a)=\epsilon_i+1$, opposite to that of x^i. We demand that the coordinates θ^i_a transform as vectors under a change of coordinates on the supermanifold M, indeed,[1]

$$\bar{x}^i=\bar{x}^i(x),\ \ \bar{\theta}^i_a=\theta^j_a\frac{\partial\bar{x}^i}{\partial x^j}\,.$$

On the supermanifold $\mathcal{M}$, one defines a tensor field of type (n,m) and rank $n+m$ as an object which in any local coordinate system (x,θ) is given by a set of functions $T^{i_1\ldots i_n}{}_{j_1\ldots j_m}(x,\theta)$, with Grassmann parity $\epsilon(T^{i_1\cdots i_n}{}_{j_1\ldots j_m}) = \epsilon(T)+\epsilon_{i_1}+\ldots+\epsilon_{i_n}+\epsilon_{j_1}+\ldots+\epsilon_{j_m}$, that transform under a change of coordinates $(x,\theta)\to(\bar{x},\bar{\theta})$ as a tensor field, of the same rank and type, defined on the supermanifold M, namely,

$$\begin{aligned}&\bar{T}^{i_1\ldots i_n}{}_{j_1\ldots j_m}\\ &=T^{l_1\ldots l_n}{}_{k_1\ldots k_m}\frac{\partial_r x^{k_m}}{\partial\bar{x}^{j_m}}\cdots\frac{\partial_r x^{k_1}}{\partial\bar{x}^{j_1}}\frac{\partial\bar{x}^{i_n}}{\partial x^{l_n}}\cdots\frac{\partial\bar{x}^{i_1}}{\partial x^{l_1}}\\ &\times(-1)^{\left(\sum_{s=1}^{m-1}\sum_{p=s+1}^{m}\epsilon_{j_p}(\epsilon_{j_s}+\epsilon_{k_s})+\sum_{s=1}^{n}\sum_{p=1}^{m}\epsilon_{j_p}(\epsilon_{i_s}+\epsilon_{l_s})\right.}\\ &\qquad\qquad{}^{\left.+\sum_{s=1}^{n-1}\sum_{p=s+1}^{n}\epsilon_{i_p}(\epsilon_{i_s}+\epsilon_{l_s})\right)}.\end{aligned}$$

Accordingly, a covariant derivative on $\mathcal{M}$ is defined as an operation $\overset{\mathcal{M}}{\nabla}_i$ that maps a tensor field of type (n,m) into a tensor field of type $(n,m+1)$ and reduces to the usual derivative $\partial_r/\partial x^i$ in a local Cartesian system on M. Explicitly, the operation $\overset{\mathcal{M}}{\nabla}_i$ has the form of a θ-extension of the covariant derivative $\overset{M}{\nabla}_i$ on the supermanifold M,

$$\overset{\mathcal{M}}{\overleftarrow{\nabla}}_i=\overset{M}{\overleftarrow{\nabla}}_i-\frac{\overleftarrow{\partial}}{\partial\theta^k_a}\theta^m_a\overset{M}{\Gamma}{}^k{}_{mi}(-1)^{\epsilon_m(\epsilon_k+1)}. \qquad (1)$$

where $\overset{M}{\nabla}_i$ maps a tensor field $T^{i_1\cdots i_n}{}_{j_1\ldots j_m}(x)$ of type (n,m) into a tensor field of type $(n,m+1)$ according to

$$\begin{aligned}&T^{i_1\cdots i_n}{}_{j_1\ldots j_m}\overset{M}{\nabla}_k=T^{i_1\cdots i_n}{}_{j_1\ldots j_m,k} \qquad (2)\\ &+\sum_{r=1}^{n}T^{i_1\ldots l\ldots j_n}{}_{j_1\ldots j_m}\overset{M}{\Gamma}{}^{i_r}{}_{lk}(-1)^{(\epsilon_{i_r}+\epsilon_l)\left(\epsilon_l+\sum_{p=r+1}^{n}\epsilon_{i_p}+\sum_{p=1}^{m}\epsilon_{j_p}\right)}\\ &-\sum_{s=1}^{m}T^{i_1\ldots i_n}{}_{j_1\ldots l\ldots j_m}\overset{M}{\Gamma}{}^l{}_{j_s k}(-1)^{(\epsilon_{j_s}+\epsilon_l)\sum_{p=s+1}^{m}\epsilon_{j_p}}.\end{aligned}$$

In (1) and (2), the functions $\overset{M}{\Gamma}{}^k{}_{ij}(x)$ are generalized Christoffel symbols (connection coefficients), having the transformation law

$$\begin{aligned}&\overset{M}{\bar{\Gamma}}{}^k{}_{ij}\\ &=(-1)^{\epsilon_j(\epsilon_m+\epsilon_i)}\frac{\partial_r\bar{x}^k}{\partial x^l}\overset{M}{\Gamma}{}^l{}_{mn}\frac{\partial_r x^n}{\partial\bar{x}^j}\frac{\partial_r x^m}{\partial\bar{x}^i}+\frac{\partial_r\bar{x}^k}{\partial x^m}\frac{\partial^2_r x^m}{\partial\bar{x}^i\partial\bar{x}^j}\,.\end{aligned}$$

In case a local Cartesian system on M does exist, the connection coefficients $\overset{M}{\Gamma}{}^k{}_{ij}(x)$ possess the property of (generalized) symmetry:

$$\overset{M}{\Gamma}{}^k{}_{ij}=(-1)^{\epsilon_i\epsilon_j}\overset{M}{\Gamma}{}^k{}_{ji}\,.$$

With this in mind, the consideration will be restricted to symmetric connections.

Since x^i and θ^i_a are independent coordinates, the expressions (1) and (2) imply that the vectors θ^i_a are covariantly constant:

$$\theta^i_a\overset{\mathcal{M}}{\nabla}_j=0. \qquad (3)$$

From (1) and (3), it follows that a (generalized) commutator $[\overset{\mathcal{M}}{\nabla}_i,\overset{\mathcal{M}}{\nabla}_j]=\overset{\mathcal{M}}{\nabla}_i\overset{\mathcal{M}}{\nabla}_j-(-1)^{\epsilon_i\epsilon_j}\overset{\mathcal{M}}{\nabla}_j\overset{\mathcal{M}}{\nabla}_i$, acting on a scalar field $T\,(x,\theta)$, can be written as

$$T[\overset{\mathcal{M}}{\nabla}_i,\overset{\mathcal{M}}{\nabla}_j]=(-1)^{\epsilon_m(\epsilon_n+1)}\frac{\partial_r T}{\partial\theta^n_a}\theta^m_a\overset{M}{R}{}^n{}_{mij}\,, \qquad (4)$$

where $\overset{M}{R}{}^i{}_{mjk}(x)$ is a curvature tensor on the supermanifold M, defined by the action of a commutator of covariant derivatives $\overset{M}{\nabla}_i$ on a vector field $T^i\,(x)$ according to

$$T^i[\overset{M}{\nabla}_j,\overset{M}{\nabla}_k]=-(-1)^{\epsilon_m(\epsilon_i+1)}T^m\overset{M}{R}{}^i{}_{mjk}\,.$$

The curvature tensor has the explicit form

$$\begin{aligned}\overset{M}{R}{}^i{}_{mjk}&=-\overset{M}{\Gamma}{}^i{}_{mj,k}+\overset{M}{\Gamma}{}^i{}_{mk,j}(-1)^{\epsilon_j\epsilon_k}\\ &+\overset{M}{\Gamma}{}^i{}_{jl}\overset{M}{\Gamma}{}^l{}_{mk}(-1)^{\epsilon_j\epsilon_m}\\ &-\overset{M}{\Gamma}{}^i{}_{kl}\overset{M}{\Gamma}{}^l{}_{mj}(-1)^{\epsilon_k(\epsilon_m+\epsilon_j)}, \qquad (5)\end{aligned}$$

and obeys a property of (generalized) antisymmetry and a (super) Jacobi identity:

$$\overset{M}{R}{}^i{}_{mjk}=-(-1)^{\epsilon_j\epsilon_k}\overset{M}{R}{}^i{}_{mkj}\,,$$

$$(-1)^{\epsilon_j\epsilon_l}\overset{M}{R}{}^i{}_{jkl}+\mathrm{cycle}\,(j,k,l)\equiv 0.$$

In what follows, we call M and $\mathcal{M}$ the base and triplectic supermanifolds, respectively, and refer to $\overset{\mathcal{M}}{\nabla}_i$ as the *triplectic covariant derivative.*

[1] In [4–6], the supermanifold $\mathcal{M}$ is parameterized by coordinates (x^i,θ_{ia}), where θ_{ia} transform as covectors, namely, $\bar{\theta}_{ia}=\theta_{ja}\frac{\partial_r x^j}{\partial\bar{x}^i}$. Instead, in the paper [10] the parameterization (x^i,θ^i_a) is used, since it is more convenient for a superfield formulation (see Sect. 2.2).

2.2 Superfield description

The complete set of variables arising in various quantization schemes [1–3] based on extended BRST symmetry can be presented as $(\phi^A, \bar{\phi}_A; \pi^A_a, \phi^*_{Aa}; \lambda^A, J_A) = (x^i, \theta^i_a, y^i)$, $i = 1, 2, \ldots, N = 2n$, $\epsilon(x^i) = \epsilon(y^i) = \epsilon_i$, $\epsilon(\theta^i_a) = \epsilon_i + 1$. This set consists of the field–antifield variables $(\phi^A, \bar{\phi}_A, \phi^*_{Aa})$, Lagrange multipliers (π^A_a, λ^A), and sources J_A to the fields. In the superfield formulation [9] of extended BRST symmetry, the variables (x^i, θ^i_a, y^i) are regarded as components of superfields $z^i(\eta)$ in a superspace with Grassmann coordinates η_a,

$$z^i(\eta) = x^i + \eta^a\theta^i_a + \eta^2 y^i, \quad \eta^2 \equiv \frac{1}{2}\eta_a\eta^a\,,$$

where raising the $Sp(2)$-indices is performed with the help of the antisymmetric second-rank tensor ε^{ab}: $\eta^a = \varepsilon^{ab}\eta_b$, $\varepsilon^{ac}\varepsilon_{cb} = \delta^a_b$.

Let us identify the components (x^i, θ^i_a, y^i) with local coordinates on a supermanifold $\mathcal{N}$, $\dim\mathcal{N} = 4N$, where the submanifold with coordinates (x^i, θ^i_a) is chosen as a triplectic supermanifold. At the same time, we define the transformations of the additional coordinates y^i, that accompany the transformations $(x, \theta) \to (\bar{x}, \bar{\theta})$, to be trivial:

$$\bar{x}^i = \bar{x}^i(x), \quad \bar{\theta}^i_a = \theta^j_a\frac{\partial\bar{x}^i}{\partial x^j}\,, \quad \bar{y}^i = y^i. \tag{6}$$

By analogy with the triplectic supermanifold $\mathcal{M}$, a tensor field of type (n, m) and rank $n + m$ on the supermanifold $\mathcal{N}$ is defined as an object which in any local coordinate system (x, θ, y) is given by a set of functions $T^{i_1\ldots i_n}{}_{j_1\ldots j_m}(x, \theta, y) \equiv T^{i_1\ldots i_n}{}_{j_1\ldots j_m}(z)$ that transform as a tensor field on the base supermanifold M. With this in mind, one can define on $\mathcal{N}$ a superfield extension $\mathcal{D}_i\,(\eta)$ of the triplectic covariant derivative $\overset{\mathcal{M}}{\nabla}_i$. Namely, one introduces $\mathcal{D}_i\,(\eta)$ as an operation that maps a tensor field of type (n, m) into a tensor field of type $(n, m+1)$ and reduces in a local Cartesian system on M to the superfield derivative[2]

$$\frac{\overleftarrow{\partial}}{\partial z^i(\eta)} = \frac{\overleftarrow{\partial}}{\partial x^i}\eta^2 + \frac{\overleftarrow{\partial}}{\partial\theta^i_a}\eta_a\,,$$

defined with respect to variations $\delta z^i(\eta) = \delta x^i + \eta^a\delta\theta^i_a$ induced by $(x, \theta, y) \to (\bar{x}, \bar{\theta}, \bar{y})$. The derivative $\mathcal{D}_i\,(\eta)$ has the explicit form

$$\overleftarrow{\mathcal{D}}_i(\eta) = \overset{\mathcal{M}}{\overleftarrow{\nabla}}_i\,\eta^2 + \frac{\overleftarrow{\partial}}{\partial\theta^i_a}\,\eta_a\,, \tag{7}$$

where each term of the η-expansion transforms as a covector with respect to $(x, \theta, y) \to (\bar{x}, \bar{\theta}, \bar{y})$.

Using $\mathcal{D}_i\,(\eta)$, one can rewrite the equalities (3) and (4) in the form

$$\frac{\partial z^i}{\partial\eta^{a\prime}}\mathcal{D}_j\,(\eta'') = \delta^i_j\eta''_a, \tag{8}$$

$$T\left[\mathcal{D}_i(\eta'), \mathcal{D}_j(\eta'')\right] \tag{9}$$

$$= (-1)^{\epsilon_m(\epsilon_n+1)}(\eta')^2\,(\eta'')^2\,\frac{\partial_r\,(T\mathcal{D}_n)}{\partial\eta'_a}\frac{\partial z^m}{\partial\eta''^a}\overset{M}{R}{}^n{}_{mij}\,,$$

where $T\,(z)$ is a scalar field, and $\overset{M}{R}{}^n{}_{mij}\,(x)$ is the curvature tensor (5).

[2] As usual, we assume that $\delta T(z) = \int \mathrm{d}^2\eta\,\frac{\partial_r T}{\partial z^i(\eta)}\delta z^i(\eta)$ and $\int \mathrm{d}^2\eta = \int \mathrm{d}^2\eta\,\eta^a = 0$, $\int \mathrm{d}^2\eta\,\eta^a\eta^b = \varepsilon^{ab}$.

3 Extended superfield realization of (modified) triplectic algebra

In this section, we shall apply the above ingredients in order to construct an extended realization of the triplectic and modified triplectic algebras [2, 3]. To this end, we recall that the triplectic algebra [2] is formed by two doublets of first- and second-order operators, $\overleftarrow{V}^a$ and $\overleftarrow{\Delta}^a$, respectively, having the Grassmann parity $\epsilon(V^a) = \epsilon(\Delta^a) = 1$ and obeying the relations

$$\begin{aligned}&\Delta^{\{a}\Delta^{b\}} = 0, \quad V^{\{a}V^{b\}} = 0,\\&V^a\Delta^b + \Delta^b V^a = 0,\end{aligned} \tag{10}$$

whereas the modified triplectic algebra [3] involves an additional doublet of first-order operators $\overleftarrow{U}^a$, $\epsilon(U^a) = 1$, and has the form

$$\begin{aligned}&\Delta^{\{a}\Delta^{b\}} = 0, \quad V^{\{a}V^{b\}} = 0, \quad U^{\{a}U^{b\}} = 0,\\&V^{\{a}\Delta^{b\}} + \Delta^{\{b}V^{a\}} = 0, \quad \Delta^{\{a}U^{b\}} + U^{\{a}\Delta^{b\}} = 0,\\&U^{\{a}V^{b\}} + V^{\{a}U^{b\}} = 0.\end{aligned} \tag{11}$$

In (10) and (11), the curly brackets stand for symmetrization with respect to the enclosed indices.

Using the second-order operators Δ^a, one can define a pair of bilinear operations $(\ ,\ )^a$,

$$\begin{aligned}(F, G)^a = {}&(-1)^{\epsilon(G)}(FG)\Delta^a\\&-(-1)^{\epsilon(G)}(F\Delta^a)G - F(G\Delta^a).\end{aligned} \tag{12}$$

which form a set of antibrackets, similar to those introduced in the $Sp(2)$-covariant formalism [1]. Thus, the operations $(\ ,\ )^a$ possess the Grassmann parity $\epsilon((F, G)^a) = \epsilon(F) + \epsilon(G) + 1$ and obey the symmetry property

$$(F, G)^a = -(-1)^{(\epsilon(G)+1)(\epsilon(F)+1)}(G, F)^a,$$

as well as the Leibniz rules

$$\begin{aligned}&(F, GH)^a = (F, G)^a H + (F, H)^a G(-1)^{\epsilon(G)\epsilon(H)},\\&(F, G)^{\{a}D^{b\}} = (F, GD^{\{a})^{b\}} - (FD^{\{a}, G)^{b\}}(-1)^{\epsilon(G)},\\&D^a \equiv \{\Delta^a, U^a, V^a\}\end{aligned} \tag{13}$$

and the Jacobi identity

$$\begin{aligned}&(F, (G, H)^{\{a})^{b\}}(-1)^{(\epsilon(F)+1)(\epsilon(H)+1)}\\&+\mathrm{cycle}(F, G, H) \equiv 0.\end{aligned} \tag{14}$$

In general coordinates, the operators (10), (11) and antibrackets (12) can be constructed [6, 10] in terms of a scalar density $\rho(x)$ and tensor fields $\omega_{ij}(x)$, $g_{ij}(x)$ defined on the base supermanifold M. At the same time, within the superfield description [10] the objects ρ, ω_{ij}, g_{ij} are identified with some functions $\mathcal{R}$, Ω_{ij}, G_{ij} formally defined on the larger supermanifold $\mathcal{N}$. In contrast to the treatment of [10], we shall present a superfield realization of (10)–(12) in terms of extended counterparts of ρ, ω_{ij}, g_{ij}, inherently defined on the supermanifold $\mathcal{N}$. The corresponding quantization procedure then follows the approach [10].

3.1 Extended realization

Let us equip the supermanifold $\mathcal{N}$ with an even scalar density $\mathcal{R}(z)$, as well as with even tensor fields $G_{ij}(z)$ and $\Omega_{ij}(z)$, the latter having the inverse[3] $\Omega^{ij}(z)$, $\epsilon(G_{ij}) = \epsilon(\Omega_{ij}) = \epsilon(\Omega^{ij}) = \epsilon_i + \epsilon_j$,

$$\Omega_{ik}\Omega^{kj}(-1)^{\epsilon_i} = \delta_i^j, \quad \Omega^{ik}\Omega_{kj}(-1)^{\epsilon_k} = \delta_j^i. \qquad (15)$$

We demand that the fields $G_{ij}(z)$ and $\Omega_{ij}(z)$, $\Omega^{ij}(z)$ obey the following properties of generalized (anti)symmetry:

$$G_{ij} = (-1)^{\epsilon_i\epsilon_j}G_{ji},$$
$$\Omega_{ij} = -(-1)^{\epsilon_i\epsilon_j}\Omega_{ji} \Leftrightarrow \Omega^{ij} = -(-1)^{\epsilon_i\epsilon_j}\Omega^{ji}.$$

At the same time, we require that $\Omega_{ij}(z)$ and $\Omega^{ij}(z)$ be covariantly constant with respect to the superfield derivative $\mathcal{D}_i(\eta)$, namely,

$$\Omega_{ij}\mathcal{D}_k = 0 \Leftrightarrow \Omega^{ij}\mathcal{D}_k = 0. \qquad (16)$$

Using the objects $\mathcal{R}(z)$, $\Omega^{ij}(z)$ and the derivative $\mathcal{D}_i(\eta)$, one can construct a superfield $Sp(2)$-doublet Δ^a of odd second-order differential operators, acting as scalars on the supermanifold $\mathcal{N}$,

$$\overleftarrow{\Delta}^a = \int \mathrm{d}^2\eta\,\eta^2 \frac{\partial_r\overleftarrow{\mathcal{D}}^i}{\partial\eta_a}\frac{\partial}{\partial\eta^2}\left(\overleftarrow{\mathcal{D}}_i + \frac{1}{2}(\mathcal{R}\overleftarrow{\mathcal{D}}_i)\right), \qquad (17)$$

where $\mathcal{D}^i(\eta)$ is a superfield derivative,

$$\mathcal{D}^i = \mathcal{D}_j\Omega^{ji} \Leftrightarrow \mathcal{D}_i = \mathcal{D}^j\Omega_{ji}(-1)^{\epsilon_j},$$

which transforms as a vector on $\mathcal{N}$. In accordance with (12), the operators (17) generate a doublet of superfield bracket operations:

$$(F,G)^a = -\int \mathrm{d}^2\eta\,\eta^2 \frac{\partial\left(F\mathcal{D}^i\right)}{\partial\eta^2}\frac{\partial\left(G\mathcal{D}_i\right)}{\partial\eta_a}(-1)^{\epsilon_i\epsilon(G)} - (-1)^{(\epsilon(F)+1)(\epsilon(G)+1)}(F\leftrightarrow G). \qquad (18)$$

Using the fields $G_{ij}(z)$ and $\Omega_{ij}(z)$, one can also equip the supermanifold $\mathcal{N}$ with the superfield objects S_0 and S_{ab}

$$S_0 = \frac{1}{2}\int \mathrm{d}^2\eta\,\eta^2 \frac{\partial_r z^i}{\partial\eta_a}G_{ij}\frac{\partial_r z^j}{\partial\eta^a}, \quad \epsilon(S_0) = 0,$$
$$S_{ab} = -\frac{1}{6}\int \mathrm{d}^2\eta\,\eta^2 \frac{\partial_r z^i}{\partial\eta^a}\Omega_{ij}\frac{\partial_r z^j}{\partial\eta^b}, \quad \epsilon(S_{ab}) = 0, \qquad (19)$$

invariant[4] under local coordinate transformations, $\bar{S}_0 = S_0$, $\bar{S}_{ab} = S_{ab}$. Here, S_0 is an $Sp(2)$-scalar, whereas S_{ab} is an $Sp(2)$-tensor, symmetric with respect to its indices, $S_{ab} = S_{ba}$.

Using S_0, S_{ab} and the bracket operations $(\;,\;)^a$, we define the $Sp(2)$-doublets of first-order odd differential operators V^a and U^a:

$$\overleftarrow{V}_a = (\,\cdot, S_{ab})^b = -\frac{1}{2}\int \mathrm{d}^2\eta\,\eta^2 \frac{\partial\overleftarrow{\mathcal{D}}_i}{\partial\eta^2}\frac{\partial_r z^i}{\partial\eta^a}, \qquad (20)$$

$$\overleftarrow{U}^a = (\,\cdot, S_0)^a = \int \mathrm{d}^2\eta\,\eta^2 \left\{\frac{\partial\overleftarrow{\mathcal{D}}^i}{\partial\eta^2}\left[G_{ij}\frac{\partial z^j}{\partial\eta_a}(-1)^{\epsilon_i} + \frac{1}{2}\frac{\partial_r z^j}{\partial\eta_b}\frac{\partial_r\left(G_{jk}\overleftarrow{\mathcal{D}}_i\right)}{\partial\eta_a}\frac{\partial z^k}{\partial\eta^b}(-1)^{\epsilon_i\epsilon_k}\right] + \frac{1}{2}\frac{\partial_r\overleftarrow{\mathcal{D}}^i}{\partial\eta_a}\frac{\partial_r z^j}{\partial\eta^b}\frac{\partial\left(G_{jk}\overleftarrow{\mathcal{D}}_i\right)}{\partial\eta^2}\frac{\partial_r z^k}{\partial\eta_b}(-1)^{\epsilon_i(\epsilon_k+1)}\right\}. \qquad (21)$$

The objects in (17)–(20) are formally identical with those arising in [10]. At the same time, the doublet of first-order operators (21) is an extension of its counterpart from [10]; namely, it contains an extra term with the expression $\partial_r\left(G_{ij}\mathcal{D}_k\right)/\partial\eta_a$, related to the additional variables present in G_{ij}.

By straightforward calculations (see Appendix A), taking into account the expressions (17) and (20) for Δ^a, V^a and the properties (8) and (9) of the derivatives $\mathcal{D}_i(\eta)$, one can show that the triplectic algebra (10) is fulfilled if the scalar density $\mathcal{R}(z)$ is chosen as

$$\mathcal{R} = -\log\,\mathrm{sdet}\,\left(\Omega^{ij}\right), \qquad (22)$$

while the tensor fields $\Omega_{ij}(z)$ and $\Omega^{ij}(z)$ obey the Jacobi identities

$$\frac{\partial_r\Omega_{ij}}{\partial z^k}(-1)^{\epsilon_i\epsilon_k} + \mathrm{cycle}\,(i,j,k) = 0$$
$$\Leftrightarrow \Omega^{il}\frac{\partial\Omega^{jk}}{\partial z^l}(-1)^{\epsilon_i\epsilon_k} + \mathrm{cycle}\,(i,j,k) = 0, \qquad (23)$$

[3] In the supersymmteric case, the contraction rules for tensor indices as well as the definition of a non-degenerate tensor can be found in the monograph [12] and papers [6, 8].

[4] Despite the fact that the transformation law (6) for the coordinates x^i, θ^i_a, y^i obviously does not result in a covariant transformation for the superfields $z^i(\eta)$, the objects S_0 and S_{ab} are scalars under (6) due to the presence of the multiplier η^2 in the corresponding integrands (19), which cancels the non-covariant contributions from $\frac{\partial z^i}{\partial\eta_a}$.

and the curvature tensor (5) of the base supermanifold M is zero:

$$\overset{M}{R}{}^{i}{}_{mjk} = 0. \qquad (24)$$

Now that the triplectic algebra (10) and the related antibrackets (12) have been explicitly constructed, the Leibniz rules (13) and the Jacobi identity (14) are obviously fulfilled. Due to (10), in order to complete the construction of the modified triplectic algebra (11), it remains to ensure the fulfillment of the relations involving the operators U^a. In this respect, the definition (21), namely, $\overleftarrow{U}^a = (\,\cdot, S_0)^a$, as well as the just mentioned properties (13) and (14), imply that the modified triplectic algebra (11) holds true in case the function S_0 is subject to

$$(S_0, S_0)^a = 0, \quad S_0 V^a = 0, \quad S_0 \Delta^a = 0. \qquad (25)$$

The consistency of these equations is implied by the properties (10) and (12) of V^a, Δ^a and $(\,,\,)^a$, which are encoded by the conditions (15), (16), (22), (23) and (24), imposed on $\mathcal{R}$, Ω_{ij} and on the base supermanifold M. The geometric interpretation of conditions (23) will be given in the following subsection, with allowance for (16) and (24). Returning to (25), we note that they always possess nonvanishing solutions (see Appendix B); however, since the following treatment does not require any special choice of such solutions, we merely assume that the relations (25) are fulfilled.

3.2 Quantization rules

Having constructed an explicit realization of the differential operators Δ^a, V^a, U^a and antibrackets $(\,,\,)^a$, we are in a position to set up a quantization procedure. This procedure repeats the BRST–antiBRST superfield covariant scheme in general coordinates [10] and has the same features. Thus, the vacuum functional is defined as

$$Z = \int \mathrm{d}z\, \mathcal{D}_0 \exp\{(\mathrm{i}/\hbar)[W + X + \alpha S_0]\}, \qquad (26)$$

with α being an arbitrary constant and the function S_0 given by (19). The quantum action $W = W(z)$ and the gauge-fixing functional $X = X(z)$ obey the quantum master equations

$$\frac{1}{2}(W, W)^a + W\mathcal{V}^a = \mathrm{i}\hbar W \Delta^a,$$
$$\frac{1}{2}(X, X)^a + X\mathcal{U}^a = \mathrm{i}\hbar X \Delta^a. \qquad (27)$$

Integration in (26) goes over the components of supervariables, $\mathrm{d}z = \mathrm{d}x\, \mathrm{d}\theta_a\, \mathrm{d}y$, and the integration measure $\mathcal{D}_0$ reads

$$\mathcal{D}_0 = \left[\mathrm{sdet}\left(\Omega^{ij}\right)\right]^{-3/2}. \qquad (28)$$

In (27), we use the operators

$$\mathcal{V}^a = \frac{1}{2}\left(\alpha U^a + \beta V^a + \gamma U^a\right),$$

$$\mathcal{U}^a = \frac{1}{2}\left(\alpha U^a - \beta V^a - \gamma U^a\right).$$

with the properties

$$\mathcal{V}^{\{a}\mathcal{V}^{b\}} = 0, \quad \mathcal{U}^{\{a}\mathcal{U}^{b\}} = 0, \quad \mathcal{V}^{\{a}\mathcal{U}^{b\}} + \mathcal{U}^{\{a}\mathcal{V}^{b\}} = 0,$$

that hold true for arbitrary values of the constant parameters α, β, γ, which implies that the operators Δ^a, $\mathcal{V}^a$, $\mathcal{U}^a$ also realize the modified triplectic algebra.

The integrand of (26) is invariant under extended BRST transformations, with the generators

$$\delta^a = (\,\cdot, W - X)^a + \mathcal{V}^a - \mathcal{U}^a,$$

which implies the independence of the vacuum functional (26) from a choice of the gauge-fixing function X (for arbitrary α, β, γ).

Let us establish a relation between [10] and the present quantization scheme in more detail. To this end, we introduce the notation

$$\Omega_{ij}(z) \equiv \omega_{ij}(x, \theta, y), \quad G_{ij}(z) \equiv g_{ij}(x, \theta, y),$$
$$\mathcal{R}(z) \equiv \rho(x, \theta, y),$$

and examine the special case $\omega_{ij} = \omega_{ij}(x)$, $g = g_{ij}(x)$, which, in view of (22), implies $\rho = \rho(x)$, so that the objects ω_{ij}, g_{ij}, ρ are restricted to the base supermanifold M, as in the formalism [10].

From (16) and (23), it follows that $\omega_{ij}(x)$ and its inverse $\omega^{ij}(x)$ are subject to

$$\omega_{ij}\overset{M}{\nabla}_k = 0 \Leftrightarrow \omega^{ij}\overset{M}{\nabla}_k = 0, \qquad (29)$$
$$\omega_{ij,k}(-1)^{\epsilon_i\epsilon_k} + \mathrm{cycle}\,(i,j,k) = 0$$
$$\Leftrightarrow \omega^{il}\partial_l\omega^{jk}(-1)^{\epsilon_i\epsilon_k} + \mathrm{cycle}\,(i,j,k) = 0, \qquad (30)$$

and therefore $\omega_{ij}(x)$, $\omega^{ij}(x)$ are identical to the antisymmetric fields of [10]. Geometrically, (30) implies that $\omega_{ij}(x)$ and $\omega^{ij}(x)$ equip the base supermanifold M with an even symplectic structure and with a Poisson bracket, respectively [6]. At the same time, (29) ensures the covariant constancy of the even differential two-form $\omega = \omega_{ij}dx^j \wedge dx^i$, so that M is interpreted as an even Fedosov supermanifold [6,8], i.e., an extension of Fedosov manifolds [7,14] to the supersymmetric case. We also observe that due to the subsidiary condition (24) the Fedosov supermanifold M is flat, as in the case of [10].

In the case $\omega_{ij} = \omega_{ij}(x)$, $g_{ij} = g_{ij}(x)$, the equalities (17)–(22) and (28) imply that the functions S_0, $\mathcal{D}_0$, the operators Δ^a, V^a, U^a and the antibrackets $(\,,\,)^a$ are reduced to the corresponding ingredients of [10]. In this respect, we note that U^a are reduced to their counterparts of [10] due to the equality

$$\int \mathrm{d}^2\eta\, \eta^2 f(\eta) \frac{\partial\left(G_{ij}\mathcal{D}_k\right)}{\partial\eta_a} = 0,$$

which holds for $g_{ij} = g_{ij}(x)$, with an arbitrary function $f(\eta)$. Consequently, in the case of $\omega_{ij}(x)$, $g_{ij}(x)$, (27) are identical with the master equations of [10], and thus (26) reduces to the vacuum functional of [10].

4 Conclusion

We have presented a natural extension of the recently proposed BRST–antiBRST superfield covariant scheme in general coordinates [10]. Thus, the coordinate dependence of the basic ingredients of [10], being the scalar density ρ and tensor fields ω_{ij}, g_{ij}, has been extended to the complete set of supervariables used in that formalism. In terms of the extended objects, $\mathcal{R}$, Ω_{ij}, G_{ij}, we have explicitly realized the differential operators Δ^a, V^a, U^a, subject to the modified triplectic algebra, and constructed the related antibracket operations $(\,,\,)^a$. The corresponding quantization procedure follows [10] and has the same general features. Thus, the formalism contains free parameters (α, β, γ), whose arbitrary choice yields a gauge-independent vacuum functional and, consequently, a gauge-independent S-matrix [15]. In the limit when the extended objects $\mathcal{R}$, Ω_{ij}, G_{ij} are reduced to the original ingredients of [10], namely, ρ, ω_{ij}, g_{ij}, the present quantization scheme becomes identical with the approach [10], and, therefore, reproduces (for a specific choice of the parameters α, β, γ in Darboux coordinates [6,10]) the previously known schemes [1–3] of covariant BRST–antiBRST quantization. It appears interesting to combine the considerations of the present work with the ideas of the recent paper [11], which proposes to enlarge the structure of a Fedosov supermanifold to the case of superfield variables. Such an opportunity, however, is impeded by the problem of a consistent superfield extension of connection coefficients [11].

Acknowledgements. The authors are grateful to the referee for critical remarks. D.M.G. thanks the foundations FAPESP and CNPq for permanent support. P.Yu.M. and J.L.T. are grateful to FAPESP.

Appendix A: Triplectic algebra

Let us examine the algebra of the differential operators Δ^a and V^a, as acting on scalars defined on the supermanifold $\mathcal{N}$. Using the expressions (1), (2) and (7), the properties (8) and (9), and the definitions (17) and (20), we obtain

$$\begin{aligned}
&\Delta^{\{a}\Delta^{b\}}\\
&= -\int \mathrm{d}^2\eta'\mathrm{d}^2\eta'' \Big\{ \mathcal{A}_{ij}\left(\eta',\eta''\right) \frac{\partial_r \mathcal{D}^j}{\partial \eta''_a} \frac{\partial \mathcal{D}^i}{\partial \eta'_b}\\
&\quad + \frac{1}{2} \frac{\partial_r \mathcal{D}^i}{\partial \eta'^c} \varepsilon^{c\{a} \Big[\mathcal{B}_i^{jb\}}\left(\eta',\eta''\right) \frac{\partial \mathcal{D}_j}{\partial \eta''^2} + \frac{\partial \mathcal{D}_j}{\partial \eta''^2} \mathcal{B}_i^{jb\}}\left(\eta',\eta''\right)\\
&\quad - \frac{1}{2} \varepsilon^{b\}d} \frac{\partial \mathcal{D}^j}{\partial \eta''^d} \left[\mathcal{B}_{ij}\left(\eta',\eta''\right) - (-1)^{\varepsilon_i\varepsilon_j} \mathcal{B}_{ji}\left(\eta',\eta''\right) \right]\\
&\quad + \frac{1}{2} \mathcal{B}_i^{jb\}}\left(\eta',\eta''\right) \frac{\partial\left(\mathcal{R}\mathcal{D}_j\right)}{\partial \eta''^2} \Big] \Big\} \eta'^2\eta''^2,
\end{aligned}$$

$$\begin{aligned}
&V^{\{a}V^{b\}}\\
&= -\frac{1}{4} \int \mathrm{d}^2\eta'\mathrm{d}^2\eta'' \mathcal{A}_{ij}\left(\eta',\eta''\right) \frac{\partial_r z^j}{\partial \eta''_a} \frac{\partial z^i}{\partial \eta'_b} \eta'^2\eta''^2,
\end{aligned}$$

$$\begin{aligned}
&2\left(\Delta^a V^b + V^b \Delta^a\right)\\
&= -\int \mathrm{d}^2\eta'\mathrm{d}^2\eta'' \Big\{ \mathcal{A}_{ij}\left(\eta',\eta''\right) \left(\frac{1}{2}\varepsilon^{ab}\Omega^{ji} + \frac{\partial_r \mathcal{D}^i}{\partial \eta'_a} \frac{\partial z^j}{\partial \eta''_b} \right)\\
&\quad + \varepsilon^{ab} \frac{\partial \mathcal{D}_i}{\partial \eta''^2} \frac{\partial}{\partial \eta'^2} \left[\frac{\partial_r \Omega^{ij}}{\partial z^j(\eta')} + \frac{1}{2}\Omega^{ij}\left(\mathcal{R}\mathcal{D}_j(\eta')\right) \right] (-1)^{\varepsilon_j}\\
&\quad - \frac{\partial D^i}{\partial \eta'_a} \left(\frac{1}{2}\mathcal{B}_{ij}\left(\eta',\eta''\right) - \overset{M}{\Gamma}{}^k_{\ ij,k} (-1)^{\varepsilon_k(\varepsilon_i+\varepsilon_j+1)} \right.\\
&\quad \left. + \overset{M}{\Gamma}{}^k_{\ jl} \overset{M}{\Gamma}{}^l_{\ ik} (-1)^{\varepsilon_j(\varepsilon_i+\varepsilon_k)+\varepsilon_k(\varepsilon_i+1)} \right) \frac{\partial_r z^j}{\partial \eta''_b} \Big\} \eta'^2\eta''^2,
\end{aligned} \tag{A.1}$$

with the following notation:

$$\begin{aligned}
\mathcal{A}_{ij}\left(\eta',\eta''\right) &\equiv \frac{\partial}{\partial \eta'^2} \frac{\partial}{\partial \eta''^2} \left[\mathcal{D}_i(\eta'), \mathcal{D}_j(\eta'') \right],\\
\mathcal{B}_j^{ia}\left(\eta',\eta''\right) &\equiv \frac{\partial\left(\mathcal{R}\mathcal{D}_j\right)}{\partial \eta'^2} \frac{\partial_r \mathcal{D}^i}{\partial \eta''_a},\\
\mathcal{B}_{ij}\left(\eta',\eta''\right) &\equiv \frac{\partial\left(\mathcal{R}\mathcal{D}_i\right)}{\partial \eta'^2} \frac{\partial \mathcal{D}_j}{\partial \eta''^2}.
\end{aligned} \tag{A.2}$$

We now subject the function $\mathcal{R}(z)$ to the equations

$$\left(\frac{\partial_r \Omega^{ij}}{\partial z^j(\eta)} + \frac{1}{2}\Omega^{ij} \frac{\partial_r \mathcal{R}}{\partial z^j(\eta)} \right) (-1)^{\epsilon_j} = 0, \tag{A.3}$$

which, in view of (15), are equivalent to

$$\frac{\partial_r \mathcal{R}}{\partial z^i(\eta)} = 2 \frac{\partial_r \Omega^{jk}}{\partial z^k(\eta)} \Omega_{ji} (-1)^{\epsilon_j+\epsilon_k}.$$

To solve these equations, we use the consequence of the Jacobi identities (23)

$$\Omega_{kj} \frac{\partial_r \Omega^{jk}}{\partial z^i(\eta)} + 2 \frac{\partial_r \Omega^{jk}}{\partial z^k(\eta)} \Omega_{ji} (-1)^{\epsilon_j+\epsilon_k} = 0,$$

and obtain the equality

$$\frac{\partial_r \mathcal{R}}{\partial z^i(\eta)} = -\Omega_{kj} \frac{\partial_r \Omega^{jk}}{\partial z^i(\eta)}. \tag{A.4}$$

Thus, the function $\mathcal{R}(z)$ can be chosen as

$$\mathcal{R} = -\log \operatorname{sdet}\left(\Omega^{ij}\right),$$

since its variation has the form

$$\begin{aligned}
\delta\mathcal{R} &= -\log \operatorname{sdet}\left(\Omega^{ij} + \delta\Omega^{ij}\right) + \log \operatorname{sdet}\left(\Omega^{ij}\right)\\
&= -\log \operatorname{sdet}\left(\delta^i_j + (-1)^{\epsilon_i}\Omega_{ik}\delta\Omega^{kj}\right)\\
&= -\operatorname{str}\left[(-1)^{\epsilon_i}\Omega_{ik}\delta\Omega^{kj}\right]\\
&= -\Omega_{ij}\delta\Omega^{ji}.
\end{aligned} \tag{A.5}$$

From (1), (7), (16) and (A.5), it follows that

$$\mathcal{R}\mathcal{D}_i(\eta) - \frac{\partial_r \mathcal{R}}{\partial z^i(\eta)} \tag{A.6}$$

$$= \Omega_{jk} \frac{\partial_r \left(\Omega^{kj} \mathcal{D}_l\right)}{\partial \eta_a} \frac{\partial z^m}{\partial \eta^a} \eta^2 \overset{M}{\Gamma}{}^{l}{}_{mi} (-1)^{\epsilon_m(\epsilon_l+1)} = 0 \,.$$

Then, due to (2), (7), (16) and (A.4), one obtains

$$\begin{aligned} &\frac{1}{2} \frac{\partial \left(\mathcal{R}\mathcal{D}_i\right)}{\partial \eta^2} \\ &= -\frac{1}{2} \Omega_{jk} \frac{\partial}{\partial \eta^2} \Bigg\{ \Omega^{kj} \mathcal{D}_i \\ &\quad -\eta^2 \Bigg[\frac{\partial_r \left(\Omega^{kj} \mathcal{D}_m\right)}{\partial \eta_a} \frac{\partial z^n}{\partial \eta^a} \overset{M}{\Gamma}{}^{m}{}_{ni} (-1)^{\epsilon_n(\epsilon_m+1)} \\ &\quad -\Omega^{km} \overset{M}{\Gamma}{}^{j}{}_{mi} (-1)^{\epsilon_m(\epsilon_j+1)} \\ &\quad - \Omega^{mj} \overset{M}{\Gamma}{}^{k}{}_{mi} (-1)^{\epsilon_m(\epsilon_j+\epsilon_k+1)+\epsilon_j\epsilon_k} \Bigg] \Bigg\} \\ &= \overset{M}{\Gamma}{}^{j}{}_{ji} (-1)^{\epsilon_j} . \end{aligned} \tag{A.7}$$

In view of (A.2), the properties (A.6) and (A.7) imply

$$\begin{aligned} &\mathcal{B}^{ia}_{j} \left(\eta', \eta''\right) \eta''^2 = 0, \\ &\mathcal{B}_{ij} \left(\eta', \eta''\right) - (-1)^{\varepsilon_i \varepsilon_j} \mathcal{B}_{ji} \left(\eta', \eta''\right) = 0, \\ &\mathcal{B}_{ij} \left(\eta', \eta''\right) = 2 \left(\overset{M}{\Gamma}{}^{k}_{ki,j} - \overset{M}{\Gamma}{}^{k}_{kl} \overset{M}{\Gamma}{}^{l}_{ij} \right) (-1)^{\epsilon_k} . \end{aligned} \tag{A.8}$$

Now, with the help of (5), (A.3) and (A.6)–(A.8), the expressions for $\Delta^{\{a}\Delta^{b\}}$ and $\Delta^a V^b + V^b \Delta^a$ in (A.1) can be written as follows:

$$\begin{aligned} &\Delta^{\{a}\Delta^{b\}} \\ &= -\int \mathrm{d}^2\eta' \mathrm{d}^2\eta'' \mathcal{A}_{ij} \left(\eta', \eta''\right) \frac{\partial_r \mathcal{D}^j}{\partial \eta''_a} \frac{\partial \mathcal{D}^i}{\partial \eta'_b} \eta'^2 \eta''^2, \\ &2 \left(\Delta^a V^b + V^b \Delta^a\right) \\ &= -\int \mathrm{d}^2\eta' \mathrm{d}^2\eta'' \mathcal{A}_{ij} \left(\eta', \eta''\right) \\ &\quad \times \left(\frac{1}{2} \varepsilon^{ab} \Omega^{ji} + \frac{\partial_r \mathcal{D}^i}{\partial \eta'_a} \frac{\partial z^j}{\partial \eta''_b} \right) \eta'^2 \eta''^2 \\ &\quad + \int \mathrm{d}^2\eta' \mathrm{d}^2\eta'' \frac{\partial \mathcal{D}^i}{\partial \eta'_a} \overset{M}{R}{}^{k}{}_{ikj} \frac{\partial_r z^j}{\partial \eta''_b} \eta'^2 \eta''^2 (-1)^{\epsilon_k(\epsilon_i+1)} . \end{aligned} \tag{A.9}$$

We recall that the commutator of derivatives $\mathcal{D}_i(\eta)$, acting on scalars defined on the supermanifold $\mathcal{N}$, is given by (9). Then, due to (A.1) and (A.9), we can see that if the base supermanifold M is chosen to be flat, $\overset{M}{R}{}^{i}{}_{jkl} = 0$, the supermanifold $\mathcal{N}$ admits an explicit realization of the triplectic algebra (10). As a consequence, the bracket operations (18) obey the properties (13) and (14), so that these operations can be interpreted as extended antibrackets.

Appendix B: Modified triplectic algebra

Let us examine the existence of non-vanishing solutions to equations (25). To this end, we recall that the function S_0 has the form (19), whereas the operators Δ^a, V^a and antibrackets $(\, , \,)^a$ are given by (17), (18) and (20). Using the properties (8) and (9) of the derivatives $\mathcal{D}_i(\eta)$, we obtain the equations

$$\begin{aligned} &(S_0, S_0)^a = \int \mathrm{d}^2\eta' \mathrm{d}^2\eta'' \\ &\quad \times \frac{\partial_r z^i}{\partial \eta'^b} \left(G_{ij} \mathcal{D}^k \left(\eta''\right)\right) \frac{\partial_r z^j}{\partial \eta'_b} \frac{\partial \left(S_0 \mathcal{D}_k\right)}{\partial \eta''_a} \eta'^2 \eta''^2 (-1)^{\varepsilon_k(\varepsilon_j+1)} \\ &= 0, \\ &S_0 V^a = \frac{1}{4} \int \mathrm{d}^2\eta' \mathrm{d}^2\eta'' \\ &\quad \times \frac{\partial_r z^i}{\partial \eta'^b} \frac{\partial \left(G_{ij} \mathcal{D}_k\right)}{\partial \eta''^2} \frac{\partial_r z^j}{\partial \eta'_b} \frac{\partial z^k}{\partial \eta''_a} \eta'^2 \eta''^2 (-1)^{\varepsilon_j \varepsilon_k} = 0, \\ &S_0 \Delta^a = \int \mathrm{d}^2\eta' \mathrm{d}^2\eta'' \\ &\quad \times \Bigg\{ \frac{\partial_r z^i}{\partial \eta'_a} \frac{\partial}{\partial \eta''^2} \left[\left(G_{ij} \mathcal{D}^j\right) + \frac{1}{2} G_{ij} \left(\mathcal{R}\mathcal{D}^j\right) \right] \\ &\quad + \frac{1}{2} \frac{\partial_r z^i}{\partial \eta'_b} \frac{\partial_r \left(G_{ij} \mathcal{D}^k\right)}{\partial \eta''_a} \frac{\partial z^j}{\partial \eta'^b} \frac{\partial}{\partial \eta''^2} \\ &\quad \times \left[\mathcal{D}_k + \frac{1}{2} \left(\mathcal{R}\mathcal{D}_k\right) \right] (-1)^{\varepsilon_k(\varepsilon_j+1)} \Bigg\} \eta'^2 \eta''^2 = 0. \end{aligned} \tag{B.1}$$

In view of the complexity of these relations, it is natural to examine some simplified restrictions that ensure the fulfillment of (B.1). For example, imposing the condition of covariance

$$G_{ij} \mathcal{D}_k = 0 \Leftrightarrow G_{ij} \mathcal{D}^k = 0 \tag{B.2}$$

and the subsidiary condition

$$G_{ij} \left(\mathcal{R}\mathcal{D}^j\right) = 0 \Leftrightarrow G_{ij} \frac{\partial_r \Omega^{jk}}{\partial z^k} (-1)^{\varepsilon_k} = 0, \tag{B.3}$$

we can see that (B.1) become identities. Note that (B.3) formally coincides with a subsidiary condition used in [10], whereas the equivalence in (B.3) is due to (A.4) and (A.6). The simplest example of solutions to (B.2) and (B.3) can be found in the class

$$G_{ij} = g_{ij}(y), \; \Omega_{ij} = \omega_{ij}(y),$$

where $g_{ij}(y)$ and $\omega_{ij}(y)$ are arbitrary (in particular, constant) functions of the auxiliary coordinates y^i. Another example may be found in the class of functions $G_{ij} = g_{ij}(x)$ and $\Omega_{ij} = \omega_{ij}(x)$,

$$g_{ij} \nabla_k = 0, \;\; \omega^{ij}_{,j} (-1)^{\varepsilon_j} = \zeta^i_{(\alpha)}, \;\; \alpha = 1, \ldots, m, \;\; m < 2n,$$

where $\zeta^i_{(\alpha)}$ stand for a set of zero-eigenvalue eigenvectors of a degenerate matrix g_{ij}. Since (B.2) and (B.3) are apparently not equivalent to the original set (B.1), the solutions of (B.1) are not necessarily exhausted by those of (B.2) and (B.3). For instance, in order to analyze the non-trivial case of non-degenerate solutions $G_{ij} = g_{ij}(x)$ it may be necessary to examine the original set of equations (B.1). This problem could be tackled by the formalism of normal coordinates recently used in [8] to investigate the structure of Fedosov supermanifolds.

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